Delay Performance at Congested Airports

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Introduction into delays at airports

- delays happen at all airports
- delays are apparent to every passenger
- delay statistics have appeared in several publications
- delays are often measured
  - in average minutes per aircraft
  - as service level (percentages of aircraft delayed more than a specified amount of time)
- there are a number of reasons for delays

Sources of delay

Processes at airports

- flight operations (arriving and departing aircraft)
  bottleneck: runway
- departing aircraft
  bottleneck: delayed feeder flights and resulting delayed passenger transfer from gate to gate
- passenger services (check-in)
  bottleneck: check-in counter, security check
- immigration
  bottleneck: immigration desk

Delays in Operations Management decisions

Operation Management problems (Barnhart et al. (2003)):

- staff planning: check-in, immigration office
  (Dowling et al. (1997), Littler and D. (1997))
- scheduling of gate operations (ramp services)
  (Bolat (2000), Andersson et al. (2003))
- capacity planning for runway systems: queueing diagrams and stochastic models
  (Newell (1979), Peterson et al. (1995))
- slot allocation and congestion pricing
  (Daniel (1995), Niemeier (2003), Forsyth and Niemeier (2004), Farkas (2000))

The classic ASP: Model formulation

Indices and parameters:

$i = 1, \ldots, P;$	planes
T_i	target landing time
S_{ij}	required separation time between planes i and j
h_i	penalty cost per unit of delay time

Continuous decision variables:

x_i	assigned landing time for plane i
β_i	delay of plane i;

Binary decision variables:

$$\delta_{ij} = \begin{cases} 1 & \text{if plane } i \text{ lands before plane } j \\ 0 & \text{otherwise} \end{cases}$$

subject to the constraints

$$
\begin{aligned}
x_i &\geq T_i & i \in \{1, \ldots, P\}; \quad (1)\\
\delta_{ij} + \delta_{ji} &= 1 & i, j \in \{1, \ldots, P\} \text{ and } i \neq j; \quad (2)\\
x_i + S_{ij} &\leq x_j + M\delta_{ji} & i, j \in \{1, \ldots, P\} \text{ and } i \neq j; \quad (3)\\
x_i - T_i &= \beta_i & i \in \{1, \ldots, P\}; \quad (4)
\end{aligned}
$$

(Bianco et al. (1999), Beasley et al. (2000), Beasley et al. (2004))

The classic ASP: Review of objective functions

Objective functions: *Minimize*

1. average wait per aircraft ($\Leftrightarrow$ total wait for all aircraft)

$$\min \sum_{i=1}^{P} \frac{\beta_i}{P} \Longleftrightarrow \min \sum_{i=1}^{P} \beta_i \tag{5}$$

2. average wait per passenger ($\Leftrightarrow$ total wait for all passengers)

$$\min \sum_{i=1}^{P} \frac{h_i \beta_i}{\sum_{j=1}^{P} h_j} \Longleftrightarrow \min \sum_{i=1}^{P} h_i \beta_i \tag{6}$$

3. cost function which is linear related to the deviation from target times

4. extension to piecewise linear functions (breakpoints)

$$\min \sum_{i=1}^{P} (h_i \beta_i + \overline{h}_i \overline{\beta}_i) \tag{7}$$

5. non-linear cost functions

$$\min \sum_{i=1}^{P} h_i(\beta_i) \tag{8}$$

6. latest landing time (makespan)

$$\min \max_{i \in \{1,\dots,P\}} \{x_i\} \tag{9}$$

The classic ASP: Numerical example

Problem: Small problem with $S_{ij} = 1$:

Objective function: minimize total delay

Optimal solutions:

i	0	1	2	3	4	5	6	7	8	9	10	$\sum_{i=1}^{P} \beta_i$	$\max_{i \in \{1,\ldots,P\}} \{\beta_i\}$
T_i	0	0	1	2	3	4	5	6	7	8	9		
$\widehat{x}_i$	0	1	2	3	4	5	6	7	8	9	10		
$\widehat{\beta}_i$		1	1	1	1	1	1	1	1	1	1	10	**1**
x_i	10	0	1	2	3	4	5	6	7	8	9		
β_i	10											10	**10**

The classic ASP: Example of Bianco et al. (1999)

			FCFS		Cheapest Insertion Heuristic (CIH)				Fair Solution			
i	Cat.	T_i	x_i	β_i	sequence	Cat.	x_i	β_i	sequence	Cat.	x_i	β_i
1	1	0	0	0	1	1	0	0	1	1	0	0
2	1	79	96	17	2	1	96	17	2	1	96	17
3	2	144	296	152	5	2	296	32	3	2	296	152
4	2	204	376	172	4	2	376	172	4	2	376	172
5	2	264	456	192	3	2	456	312	5	2	456	192
6	1	320	528	208	7	1	528	0	6	1	528	208
7	1	528	624	96	6	1	624	304	7	1	624	96
8	1	635	720	85	8	1	720	85	8	1	720	85
9	2	730	920	190	10	1	816	50	10	1	816	50
10	1	766	992	226	12	1	920	0	12	1	920	0
11	2	790	1192	402	14	2	1120	14	9	2	1120	390
12	1	920	1264	344	16	2	1200	34	11	2	1200	410
13	2	1046	1464	418	17	2	1280	54	13	2	1280	234
14	2	1106	1544	438	19	2	1360	74	14	2	1360	254
15	1	1136	1616	480	20	2	1440	22	16	2	1440	274
16	2	1166	1816	650	13	2	1520	474	17	2	1520	294
17	2	1226	1896	670	11	2	1600	810	19	2	1600	314
18	1	1233	1968	735	9	2	1680	950	20	2	1680	262

			FCFS		Cheapest Insertion Heuristic (CIH)				Fair Solution			
i	Cat.	T_i	x_i	β_i	sequence	Cat.	x_i	β_i	sequence	Cat.	x_i	β_i
19	2	1286	2168	882	23	2	1760	11	23	2	1760	11
20	2	1418	2248	830	25	2	1840	31	25	2	1840	31
21	1	1642	2320	678	26	2	1920	51	26	2	1920	51
22	1	1715	2416	701	28	2	2000	11	27	2	2000	71
23	2	1749	2616	867	27	2	2080	151	28	2	2080	91
24	1	1770	2688	918	29	1	2152	78	15	1	2152	1016
25	2	1809	2888	1079	30	1	2248	80	18	1	2248	1015
26	2	1869	2968	1099	32	1	2344	85	21	1	2344	702
27	2	1929	3048	1119	34	1	2440	13	22	1	2440	725
28	2	1989	3128	1139	35	1	2536	55	24	1	2536	766
29	1	2074	3200	1126	24	1	2632	862	29	1	2632	558
30	1	2168	3296	1128	39	1	2728	49	30	1	2728	560
31	2	2229	3496	1267	22	1	2824	1109	32	1	2824	565
32	1	2259	3568	1309	40	1	2920	37	34	1	2920	493
33	2	2326	3768	1442	41	1	3016	34	35	1	3016	535
34	1	2427	3840	1413	43	1	3112	21	39	1	3112	433
35	1	2481	3936	1455	42	1	3208	162	40	1	3208	325
36	2	2488	4136	1648	21	1	3304	1662	41	1	3304	322
37	2	2565	4216	1651	15	1	3400	**2264**	42	1	3400	354
38	2	2657	4296	1639	18	1	3496	2263	43	1	3496	405
39	1	2679	4368	1689	44	2	3696	543	31	2	3696	**1467**
40	1	2883	4464	1581	38	2	3776	1119	33	2	3776	1450
41	1	2982	4560	1578	37	2	3856	1291	36	2	3856	1368
42	1	3046	4656	1610	36	2	3936	1448	37	2	3936	1371
43	1	3091	4752	1661	33	2	4016	1690	38	2	4016	1359
44	2	3153	4952	**1799**	31	2	4096	1867	44	2	4096	943

	FCFS	Cheapest Insertion Heuristic	Fair Solution
$\sum \beta_i$	38783	20391 (-47.4 %)	20391 (-47.4 %)
$\overline{\beta} = \sum \beta_i / I$	881.4	463.4 (-47.4 %)	463.4 (-47.4 %)
max$\{\beta_i\}$	1799	2264 (+25.8 %)	1467 (-18.5%)
max$\{x_i\}$	4952	4096 (-17.3%)	4096 (-17.3%)

The optimization of the sequence according to average delays

- may rise to long waiting times for some aircraft,
- may violate fairness condition,
- hence, it has to be improved.

Reformulations of the ASP

Three ways to take fairness into account

- non-linear objective function
- restrict the sequence within one class to FCFS
  $\Longrightarrow$ additional fairness conditions

$$\delta_{ij} = 1 \qquad \text{if } i \text{ and } j \text{ belongs to the same category } \textbf{and } T_i \leq T_j \tag{10}$$

- reformulation with aircraft classes

Reformulation with aircraft classes: Model

Indices and parameters:

$c = 1, \ldots, C;$	aircraft classes
$P_c;$	set of planes of class c
T_{i_c}	target landing time for plane $i_c \in P_c$ (with $T_{i_c} \leq T_{i_c+1}$)
$S_{c\overline{c}}$	required separation time between landings of planes of type c and $\overline{c}$
h_c	penalty cost per unit time of delay for planes of class c

Continuous decision variables:

x_{i_c}	assigned landing time for plane $i_c \in P_c,\ c \in C$
β_{i_c}	delay of plane i_c

Binary decision variables:

$$\delta_{i_c, j_{\overline{c}}} = \begin{cases} 1 & \text{if plane } i_c \text{ lands before plane } j_{\overline{c}} \\ 0 & \text{otherwise} \end{cases} \quad \text{for } i_c \in P_c,\ j_{\overline{c}} \in P_{\overline{c}} \text{ and } c \neq \overline{c}$$

The objective is to minimize the overall weighted delay

$$\text{minimize} \sum_{c=1}^{C} \sum_{i_c \in P_c} (h_{i_c} \beta_{i_c}) \tag{11}$$

subject to the constraints

$$\begin{aligned}
x_{i_c} &\geq T_{i_c} && i_c \in P_c \text{ and } c \in \{1, \ldots, C\}; && (12)\\
\delta_{i_c, j_{\overline{c}}} + \delta_{j_{\overline{c}}, i_c} &= 1 && i_c \in P_c,\ j_{\overline{c}} \in P_{\overline{c}} \text{ and } c \neq \overline{c}; && (13)\\
x_{i_c} + S_{cc} &\leq x i_c + 1 && i_c \in \{1, \ldots, |P_c| - 1\} \text{ and } c \in \{1, \ldots, C\}; && (14)\\
x_{i_c} + S_{c\overline{c}} &\leq x_{j_{\overline{c}}} + M \delta_{i_c, j_{\overline{c}}} && i_c \in P_c,\ j_{\overline{c}} \in P_{\overline{c}} \text{ and } c \neq \overline{c}; && (15)\\
x_{i_c} - T_{i_c} &= \beta_{i_c} && i_c \in P_c \text{ and } c \in \{1, \ldots, C\}; && (16)
\end{aligned}$$

Reformulation with aircraft classes: Conclusion

- new class of scheduling problems
- hard to obtain optimal solutions by linear programming
- adaption of known heuristics to the new problem possible
- additional constraints for real world problems

Conclusion

- Take into account different delay measures (average delay vs. maximum delay).
- Optimization subject to average delays may result in unfair solutions.
- Models with fairness conditions are hard to optimize.
- Development of **new** heuristic algorithms is interesting and necessary.

Thank you for your attention!

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