Game theoretic analysis of price competition in cargo industries

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joint work with
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Overview

- Motivation
- The market model
- Nash equilibrium
- Results

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The market

- Every company can buy a single plane with capacity 1 for a fix cost F
- There exist “sufficiently” many companies
- The demand is linearly decreasing $D(p)=(a-p)/(2b)$
- Every company sets a price p_i. If the plane transports x_i goods units then revenue: p_i*x_i
- Planes with lower prices are chartered first. Customers with highest willingness to pay are served first
- If several liners set the same price then the demand is distributed evenly

Two models

Every company tries to maximise its profit.
In particular, companies enter the market as long as they expect to cover fix cost

- Free market
  No cooperation
- Conference and independents
  Some companies join a conference and set a common price. Then independents choose their prices

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Nash equilibrium

Informal definition: A constellation in which every company cannot increase its revenue by changing its behaviour

Assumption on perfect information: All that can be possibly deduced within the game is known by the players. Not realistic for chess, e.g.

Extra condition subgame-perfect: No unrealistic threats

Pure and mixed strategies

A pure strategy is a unique behaviour

A mixed strategy is a probability distribution over various behaviours

If a teacher only occasionally controls the homework then he will use a mixed strategy

We consider pure equilibria and symmetric mixed eq

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The Nash-equilibrium of the game

Let $M=b(1+D(0)-floor(D(0)))^2/2$.
Let N = number of market entrants

Case A: If $F>M$ then $N=floor(D(F))$ and they set the market clearing price for their total capacity. This is a pure pricing strategy

Case B: If $F<M$ then $N=D(0)$ and they use a mixed pricing strategy

Let $F>2b$ from now on. Then only case A can occur

Welfare maximisation

Utility for x goods units being transported is

$$U(x)=ax-bx^2$$

The welfare for x goods units being transported is

$$W(x)=U(x)-\mathrm{ceil}(x)*F$$

Maximal welfare occurs for N or N+1 planes. They operate at full capacity

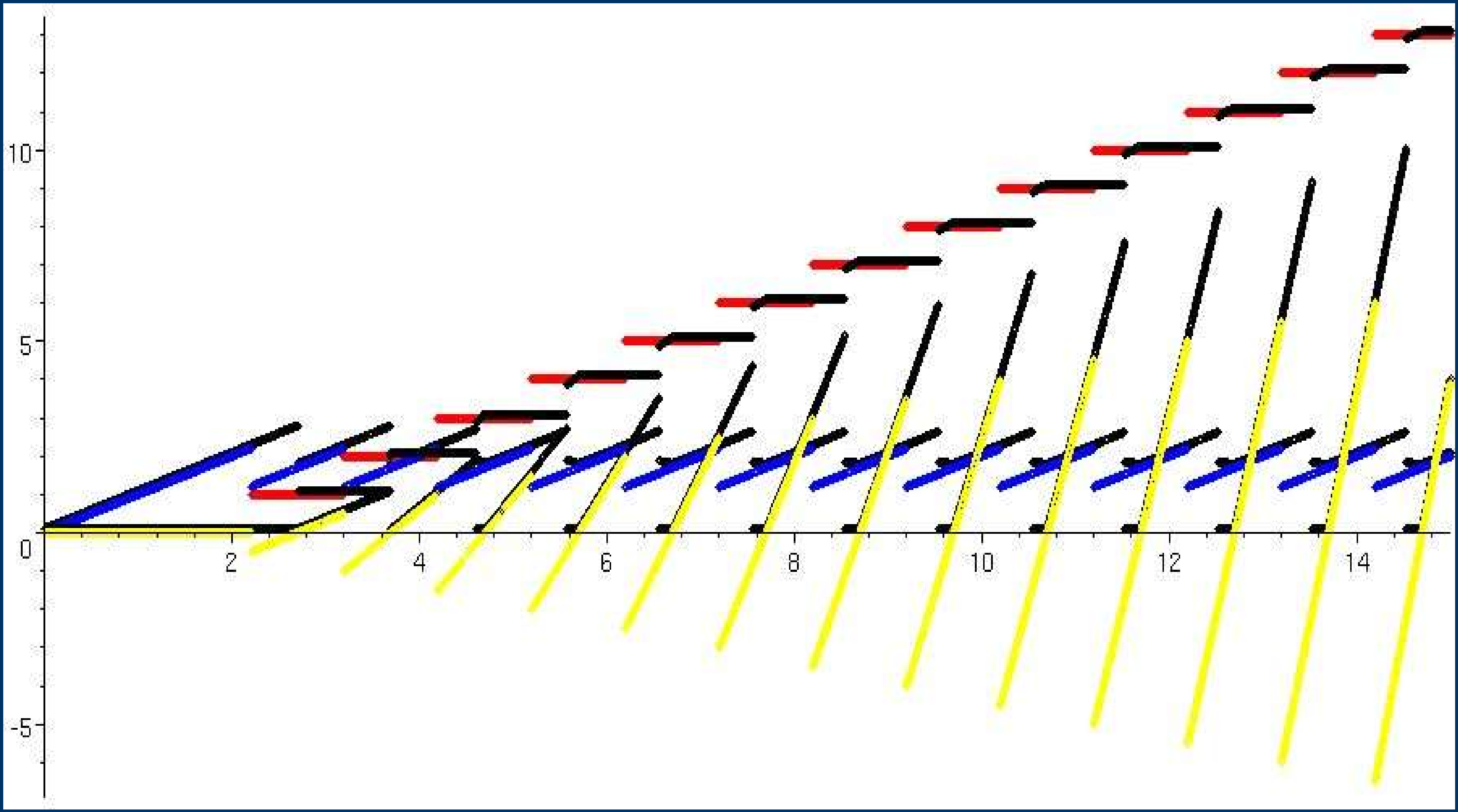
Welfare maximisation under fix cost covering

Revenue R(x)=P(x)*x=(a-2bx)*x > ceil(x)*F
where x is the amount of transported goods

If the maximal welfare occurs for N planes then fix costs are covered

If the maximal welfare occurs for N+1 planes then fix costs are not covered. Two options:

- either N planes at full capacity or
- N+1 planes with reduced capacity



Example: b=0.5, F=1.7

x-axis: D(0)

y-axis: transported amount, price, profit at welfare max

Summary

- Full analysis of this (simple) model
- The free market is stable, i.e. there always exists a Nash-equilibrium. The symmetric strategies of the market entrants may be pure or mixed
- The Nash-equilibrium is not far off the welfare maximum

Thank you for your attention